Infrared Absorption of High Temperature Superconductors

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Abstract
The temperature dependence of infrared absorption coefficients for high temperature superconductors are investigated in two forms: (i) in the close vicinity of T_c the superconductivity increases so that the cooper pairs dominate over normal electronics contribution and (ii) the contribution of normal energy becomes small compared to the cooper pair energy. The developed expressions electronic infrared absorption coefficients for high temperature superconductors are obtained by using double time temperature dependent Green's functions. The almost complete Hamiltonian used in the study include the contributions due to the unperturbed phonons and electrons, anharmonic phonon fields, localized phonon fields and that of electron-phonon interactions. The total infrared absorption in high temperature superconductors is obtained as the sum of calculations of strength of absorption at the gap- or local-mode energies have been made for some samples. The non availability of several physical quantities for Nb_3Sn compound and involvement of the wide range of energies, a great complexity in computation was eradicated by showing the general trends of the variation of absorption coefficient with changing energy and temperature.

Index Terms:- Absorption coefficient, Cooper pairs, Hamiltonian, Infrared, phononic and electronic, Impurity concentration and superconductors.

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1 Introduction

Well known properties of real crystal, e.g., lattice thermal conductivity, thermal expansion, phonon life time and dielectric properties as observed by infra-red spectroscopy which cannot be explained by harmonic approximation [1]. In the harmonic approximation the absorption spectrum of a cubic crystal reveals a sharp line at the dispersion frequency, which is the frequency of the transverse mode of zero wave vectors in the optical branch. These results are contradicted by experimental evidence which shows that, strong absorption at the dispersion frequency is superimposed on a continuous background of absorption on subsidiary maxima. The calculations of Pauli [2], this discrepancy has been attributed to neglect the anharmonic terms in the expansion of the lattice potential energy in powers of the displacements of ions from their equilibrium positions. Pauli studied a linear chain of alternating positively and negatively charged particles of equal mass and obtained a dispersion formula. Such a formula predicts a continuous absorption with maxima at the dispersion frequency; it does not predict any subsidiary maxima. Pauli's work was extended by Born and Blackman [3, 4] and studied a linear chain of alternating positively and negatively charged particles of different masses. They showed that in this model the effect of these anharmonic terms is

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to damp the infinitely sharp peaks at the dispersion frequency, to provide a continuous absorption at frequencies different from the dispersion frequency, and to provide subsidiary or secondary maxima in the spectra.

Born and Huang gave a new treatment of anharmonic optical absorption in complex lattices. It is a quantum mechanical treatment based on a modification of time-dependent perturbation theory of Weisskopf and Wigner [5]. According to this theory, high temperature range gives the damping with T^3 dependence. Mitskevich [6] has taken case of the third- and fourth-order anharmonicity. Vinogradov [7] has enumerated the deficiencies in the Born and Huang theory and used the Green's function technique to study infrared absorption. Kashcheev [8] extend this method to study the infrared absorption considering third- and fourth-order anharmonicity. Born and Huang approach, Maradudin *et. al.* and others [9,10] investigated the theory of absorption spectrum of a particular nontrivial model: a linear chain of alternating positively and negatively charged particles with different masses and nearest-

neighbor interaction only. Their results are comparable directly with the Born and Huang's results with some discrepancies. The work discussed above effect of the anharmonicities on infrared absorption in crystals has not been sufficiently accurate.

The anharmonicity is described[11] for infrared absorption using temperature-dependent double-time Green's function technique and taking into account both the lattice anharmonicity and higher order electric dipole moments of the crystal. A modification of the Kubo formalism for treating transport coefficient is employed to obtain general expressions for dielectric susceptibility and absorption coefficient of crystals having arbitrary structure.

Many authors [12-15] have studied theoretically the problem of infrared absorption due to a very low concentration of impurities in the crystal. Using the temperature dependent double-time Green's functions and Kubo formulism in term of dielectric susceptibility and provided sufficient information on infrared absorption.

The infrared properties of the high temperature superconductors have attracted considerable attention [16-20] with many attempts to determine the superconducting energy gap as well as the low-frequency excitations. The microscopic origin of the gap was clarified by the theory of Bardeen *et al* [21] which also successfully explained superconductor properties in the static electromagnetic field such as Meissner effect. Furthermore, other authors considered effects of impurities [22] and strong coupling [23]. Great attention was paid to the analysis of the optical spectra of low-*Tc* and high-*Tc* superconductors in the infrared and far-infrared regions [24–27]. A complete review of the optical properties of high temperature superconductors can be found in papers [28, 29].

2 Absorption Formulism
The optical absorption coefficient related to the Green's function [17 and 30] for a light wave of energy $\in$, with velocity v in a medium of refractive index η can be written in the form In the cubic case of a diatomic crystal

$$\alpha(\in) = \left(\frac{4\pi N e^2}{\eta v V}\right)\left(\frac{M_1 + M_2}{3M_1 M_2}\right)\in \text{Im} G_j(0, \in) \qquad (1)$$

where M_1 and M_2 are the masses of two atoms, j is one of the branch indices and N describes the number of cell in the crystal. The above Eq.(1) can be arranged [17] as

$$\alpha(\in) = \left(\frac{4\pi N e^2}{\eta v}\right)\left(\frac{\pi \mu \in}{\in_0}\right)\text{Im} G(k, \in) \qquad (2)$$

where the reduced mass of a crystal is μ, $\in_0$ is the transverse optical mode energy at $k = 0$ for the host lattice. For the diatomic lattice with masses M_1 and M_2 reduces to $\mu^{-1} = M_1 M_2 (M_1 + M_2)^{-1}$. The linear infrared absorption coefficient $\kappa(\in)$ can easily be obtained on substitution of imaginary part of Green's function into Eq.(2).

3 The Hamiltonian
The almost complete Hamiltonian of such a system can be written as

$$H = H_p + H_e + H_{ep} + H_A + H_D \qquad (3)$$

where H_p is the unperturbed phonon Hamiltonian, H_e is the unperturbed electron Hamiltonian, H_{ep} is the electron-phonon Hamiltonian, H_A is the anharmonic Hamiltonian and H_D is the defect Hamiltonian arising due to the substitution impurities respectively, which is given by

$$H_p = \sum_{\mathbf{k}} \frac{\in_k}{4}[A_{\mathbf{k}}^* A_{\mathbf{k}} + B_{\mathbf{k}}^* B_{\mathbf{k}}] \qquad (4a)$$

$$H_e = \sum_{\mathbf{q}} \in_{\mathbf{q}} b_{\mathbf{q}}^* b_{\mathbf{q}}, \qquad (4b)$$

$$H_{ep} = \sum_{\mathbf{k},\mathbf{q}} g_{\mathbf{k}} b_{\mathbf{Q}}^* b_{\mathbf{q}} B_{\mathbf{k}}, \qquad (4c)$$

$$H_A = \sum_{s \geq 3} \sum_{\mathbf{k}_1,...,\mathbf{k}_s} V_s(\mathbf{k}_1, \mathbf{k}_2, ..., \mathbf{k}_s) A_{\mathbf{k}_1} A_{\mathbf{k}_2} A_{\mathbf{k}_s} \qquad (4d)$$

and

$$H_D = \sum_{\mathbf{k}_1,\mathbf{k}_2}[D(\mathbf{k}_1, \mathbf{k}_2) A_{\mathbf{k}_1} A_{\mathbf{k}_2} - C(\mathbf{k}_1, \mathbf{k}_2) B_{\mathbf{k}_1} B_{\mathbf{k}_2}] \quad (4e)$$

In above expressions $A_{\mathbf{k}} = a_{\mathbf{k}} + a^*_{-\mathbf{k}} = +A^*_{-\mathbf{k}}$ (phonon field operator) and $B_{\mathbf{k}} = a_{\mathbf{k}} - a^*_{-\mathbf{k}} = -B^*_{-\mathbf{k}}$ (phonon momentum operator), $b_{\mathbf{q}}(b^*_{\mathbf{q}})$ and $a_{\mathbf{k}}(a^*_{\mathbf{k}})$ are electron and phonon annihilation (creation) operators with wave vectors $\mathbf{q}$ and $\mathbf{k}$ respectively. $\mathbf{Q} = \mathbf{k} + \mathbf{q}$. $\in_{\mathbf{k}}, \in_{\mathbf{q}}, g_{\mathbf{k}}, V_s(\mathbf{k}_1, \mathbf{k}_2, ..., \mathbf{k}_s), C(\mathbf{k}_1, \mathbf{k}_2)$ and $D(\mathbf{k}_1, \mathbf{k}_2)$ stand for the electron energy, phonon energy and electron-phonon coupling coefficient, anharmonic coefficients, mass difference and force constant change parameters, respectively.

4 Electron Green's function

Let us consider the double-time thermodynamic electron retarded Green's functions

$$G_{qq'}(t,t') = \langle\langle b^*_{q\sigma}(t); b_{q'\sigma'}(t') \rangle\rangle$$
$$= -i\theta(t-t') < [b^*_{q\sigma}, b_{q'\sigma'}] > \quad (5)$$

In above Eq.(5), σ defines the spin and $\uparrow(\downarrow)$ designates the spin up (down) for electrons.

With the help of equation of motion technique of quantum dynamics via Hamiltonian (4) in the form

$$G_{qq'}(\in) = \frac{(3\in^N + \in^C)\delta_{qq'}\delta_{\sigma\sigma'}}{\{2\pi[\in^2 - \tilde{\in}^2_{q\sigma} + (3\in^N + \in^C)P(q,\in)]\}} \quad (6)$$

where $\tilde{\in}^2_{q\sigma}$ $(\equiv \tilde{\in}^2_q)$ is the renormalized energy, which is equal to

$$\tilde{\in}^2_q = (3\in^N + \in^C)^2 - \frac{X_1}{2\pi} - \frac{1}{(3\in^N + \in^C)}$$
$$= \sum_{q'}(g_k + g^*_{k'})\frac{X_2}{2\pi} \quad (7)$$

and

$$P(q,\in) = \frac{1}{2\pi(3\in^N + \in^C)^2} \langle\langle F^*_{q\sigma}(t); F_{q'\sigma'}(t') \rangle\rangle_\in \quad (8)$$

where $\in^C$ is the energy of cooper pair. In Eq.(6) the delta function $\delta_{q\sigma}$ acquires a large number of momentum and spin combinations, namely $\delta_{q\uparrow}\delta_{q\uparrow}$, $\delta_{q\uparrow}\delta_{q\downarrow}$, $\delta_{q\downarrow}\delta_{q\uparrow}$, $\delta_{q\downarrow}\delta_{q\downarrow}$, $\delta_{-q\uparrow}\delta_{q\uparrow}$, $\delta_{-q\uparrow}\delta_{q\downarrow}$, $\delta_{-q\downarrow}\delta_{q\uparrow}$, $\delta_{-q\downarrow}\delta_{q\downarrow}$. During the above development it is surprisingly found that the cooper pair energy $\in^C$ as well as the normal electron energy $\in^N$ automatically emerges out in the results. The solution of function $P(q,\in)$ can be obtained after decoupling of the developed Green's functions from the Green's function $\langle\langle F^*_{q\sigma}(t); F_{q'\sigma'}(t') \rangle\rangle$, with the help of electron and phonon renormalized Hamiltonians

$$H^{(0)}_{ren(e)} = \sum_q (\tilde{\in}_{q\uparrow} b^*_{q\uparrow}b_{q\uparrow} + \tilde{\in}_{q\downarrow} b^*_{q\downarrow}b_{q\downarrow} + \tilde{\in}_{-q\uparrow} b^*_{-q\uparrow}b_{-q\uparrow} + \tilde{\in}_{-q\downarrow} b^*_{-q\downarrow}b_{-q\downarrow}) \quad (9)$$

$$H^{(0)}_{ren(p)} = \frac{1}{4}\sum_k [\frac{\tilde{\in}^2_k}{\in_k}A^*_k A_k + \in_k B^*_k B_k] \quad (10)$$

$$P(q,\in) = \sum_{k,k'} G_{k,k'}^2 \left\{ \left[\left(-\frac{8\,\widetilde{\in}_k^2}{\in_k} + \frac{2\in_k^3}{(3\in^N + \in^C)^2} \right) \frac{N_{Q\sigma}}{\in^2 - \widetilde{\in}_k^2} + \left(\frac{\in_k^2 n_k}{(3\in^N + \in^C)^2} \right. \right. \right.$$

$$\left. + \frac{4\in_k \tilde{n}_k}{3\in^N + \in^C} + \widetilde{\tilde{n}}_k \right] \frac{4}{\in - (3\widetilde{\in}^N + \widetilde{\in}^C)} + 128 \sum_{k_1} \left[D(k_1,-k)D(-k_1,-k') \frac{\in_{k_1} N_{Q\sigma}}{\in^2 - \widetilde{\in}_{k_1}^2} \right.$$

$$\left. + D(k_1,-k)D(k_1,-k') \frac{n_{k_1}}{\in - (3\widetilde{\in}^N + \widetilde{\in}^C)} \right] \frac{1}{(3\in^N + \in^C)^2}$$

$$+ 288 \sum_{k_1,k_2} \left[V_3(k_1,k_2,-k)V_3(-k_1,-k_2,-k') \left(\frac{S_{+\alpha}\,\widetilde{\in}_{+\alpha}}{\in^2 - \widetilde{\in}_{+\alpha}^2} + \frac{S_{-\alpha}\,\widetilde{\in}_{-\alpha}}{\in^2 - \widetilde{\in}_{-\alpha}^2} \right) \eta_1 N_{Q\sigma} \right.$$

$$\left. + V_3(k_1,k_2,-k)V_3(k_1,k_2,-k') \left(\frac{n_{k_1}n_{k_2}}{\in - (3\widetilde{\in}^N + \widetilde{\in}^C)} \right) \right] \frac{1}{(3\in^N + \in^C)^2}$$

$$+ 1536 \sum_{k_1,k_2,k_3} \left[V_4(k_1,k_2,k_3,-k)V_4(-k_1,-k_2,-k_3,-k') \left(\frac{S_{+\beta}\,\widetilde{\in}_{+\beta}}{\in^2 - \widetilde{\in}_{+\beta}^2} + 3\frac{S_{-\beta}\,\widetilde{\in}_{-\beta}}{\in^2 - \widetilde{\in}_{-\beta}^2} \right) \right.$$

As $$\times \eta_2 N_{Q\sigma} + V_4(k_1,k_2,k_3,-k)V_4(k_1,k_2,k_3,-k') \left(\frac{3n_{k_1}n_{k_2}n_{k_3}}{\in - (3\widetilde{\in}^N + \widetilde{\in}^C)} \right) \right] \frac{1}{(3\in^N + \in^C)^2} \right\} \qquad (11)$$

where

$$n_k = \langle A_k A_k \rangle \,; \tilde{n}_k = \langle A_k B_k \rangle \,; \widetilde{\tilde{n}}_k = \langle B_k B_k \rangle \qquad (12a)$$

$$G_{k,k'}^2 = (g_k + g_k^*)(g_{k'} + g_k^*) \qquad (12b)$$

$$\widetilde{\in}_{\pm\alpha} = \widetilde{\in}_{k_1} \pm \widetilde{\in}_{k_2} \,; \widetilde{\in}_{\pm\beta} = \widetilde{\in}_{k_1} \pm \widetilde{\in}_{k_2} \pm \widetilde{\in}_{k_2} \,; \qquad (12c)$$

$$S_{\pm\alpha} = n_{k_2} \pm n_{k_1} \,; S_{\pm\beta} = 1 \pm n_{k_1}n_{k_2} + n_{k_2}n_{k_3} \pm n_{k_3}n_{k_1} \qquad (12d)$$

$$\eta_1 = \frac{\in_{k_1} \in_{k_2}}{\widetilde{\in}_{k_1} \widetilde{\in}_{k_2}} \,; \; \eta_2 = \frac{\in_{k_1} \in_{k_2} \in_{k3}}{\widetilde{\in}_{k_1} \widetilde{\in}_{k_2} \widetilde{\in}_{k_3}} \qquad (12e)$$

and

$$N_{Q\sigma} = \int_{-\infty}^{+\infty} \frac{e^{-i\in(t-t')}}{e^{\beta\in} \pm 1} \delta[\in - (3\widetilde{\in}^N + \widetilde{\in}^C)]d\in \qquad (12f)$$

The value of $N_{Q\sigma}$ is including through the cooper pairs and normal electron problem via $\delta[\in - (3\in^N + \in^C)]$ function. This cannot be normally solved with usual process. Hence, here we can write above expressions as (to the reasonable degree of accuracy and without violating physical lows)

$$N_{Q\sigma} \cong \int_{-\infty}^{+\infty} e^{-i\in(t-t')} \frac{1}{2} \left[\frac{\delta(\in - 3\in^N)}{e^{\beta\in} + 1} + \frac{\delta(\in - \in^C)}{e^{\beta\in} - 1} \right] d\in \qquad (13)$$

or $$N_{Q\sigma} \cong \frac{1}{2} \left[\frac{1}{e^{3\beta\in^N} + 1} + \frac{1}{e^{\beta\in^C} - 1} \right] \qquad (t = t') \qquad (14)$$

The response function can be written as

$$P(q,\in) = \Delta_{(ep)}(q,\in) + i\Gamma_{(ep)}(q,\in) \qquad (15)$$

The electron-phonon energy shifts $\Delta_{(ep)}(q,\in)$ are the principal value of $P(q,\in)$. The shift $\Delta_{(ep)}(q,\in)$ and line width $\Gamma_{(ep)}(q,\in)$ can be separated in three terms, such as

$$\Gamma_{(ep)}(q,\in) = \Gamma_{(ep)}^{EP}(q,\in) + \Gamma_{(ep)}^D(q,\in) + \Gamma_{(ep)}^A(q,\in) \qquad (16)$$

$$\Gamma^{EP}_{(ep)}(q,\in) = \Omega_v G_k^2 \left\{ \left[-4\,\widetilde{\in}_k^2 + \frac{\widetilde{\in}_k^4}{(3\in^N + \in^C)^2} \right] N_{Q\sigma} + 2\left[\frac{\in_k^2 n_k}{(3\in^N + \in^C)^2} \right. \right.$$

$$\left. \left. + \frac{4\in_k \tilde{n}_k}{(3\in^N + \in^C)} + 4\tilde{n}_k \right](3\widetilde{\in}^N + \widetilde{\in}^C)^2 \right\} \qquad (17a)$$

$$\Gamma^{D}_{(ep)}(q,\in) = 32\Omega_v \sum_{k_1} |D(k_1,-k)|^2 G_k^2 [\in_{k_1} \widetilde{\in}_{k_1} N_{Q\sigma} + n_{k_1}(3\widetilde{\in}^N + \widetilde{\in}^C)^2] \frac{1}{(3\in^N + \in^C)^2} \qquad (17b)$$

$$\Gamma^{3A}_{(ep)}(q,\in) = 72\Omega_v \sum_{k_1,k_2} G_k^2 |V_3(k_1,k_2,-k)|^2 \eta_1 \{[S_{+\alpha}\,\widetilde{\in}_{+\alpha} + S_{-\alpha}\,\widetilde{\in}_{-\alpha}]N_{Q\sigma}$$

$$+ (3\widetilde{\in}^N + \widetilde{\in}^C)^2 n_{k_1} n_{k_2}\} \frac{1}{(3\in^N + \in^C)^2} \qquad (17c)$$

$$\Gamma^{4A}_{(ep)}(q,\in) = 384\Omega_v \sum_{k_1,k_2,k_3} G_k^2 |V_3(k_1,k_2,k_3,-k)|^2 \{\eta_2[S_{+\beta}\,\widetilde{\in}_{+\beta} + 3S_{-\beta}\,\widetilde{\in}_{-\beta}]N_{Q\sigma}$$

$$+ (3\widetilde{\in}^N + \widetilde{\in}^C)^2 n_{k_1} n_{k_2} n_{k_3}\} \frac{1}{(3\in^N + \in^C)^2} \qquad (17d)$$

Now, the imaginary part of $G^{(e)}(q,\in)$ is given by

$$G_{qq'}(\in) = \frac{(3\in^N + \in^C)^2 \delta_{qq'} \delta_{\sigma\sigma'} \Gamma_{(ep)}(q,\in)}{\{2\pi[(\in^2 - \overline{\in}_q^2)^2 + (3\in^N + \in^C)^2 [\Gamma_{(ep)}(q,\in)]^2]\}} \qquad (18)$$

5 Infrared absorption coefficient

The linear infrared absorption coefficient $\alpha_{(ep)}(\in)$ can easily be obtained on substitution of imaginary part of electron Green's function given by (18) into (2) as

$$\alpha_{(ep)}(\in) = \left(\frac{2\pi N e^2 \mu}{\eta v} \right) \frac{(3\in^N + \in^C)^2 \Gamma_{(ep)}(q,\in)}{[(\in^2 - \overline{\in}_q^2)^2 + (3\in^N + \in^C)^2 \Gamma_{(ep)}^2(q,\in)]} \qquad (19)$$

The Lorentzian line shape distribution appearing in above Eq.(19) can be simplified under reasonable approximations for the small value of $\Gamma_{(ep)}(q,\in)$ far from the fundamental absorption peak $\in = \in_q$, to give

$$\alpha_{(ep)}(\in) = \left(\frac{1}{4} \right)\left(\frac{\in}{\in_0} \right)\left[\frac{P_c(3\in^N + \in^C)^2 \Gamma(q,\in)}{(\in^2 - \overline{\in}_q^2)^2} \right] \qquad (20)$$

The electron infrared absorption coefficient $\alpha_{(ep)}(\in)$ can be separated in three terms, namely: (i) $\alpha^A_{(ep)}(\in)$ (anharmonic contribution), (ii) $\alpha^D_{(ep)}(\in)$ (defect contribution) and (iii) $\alpha^{EP}_{(ep)}(\in)$ (electron-phonon contribution). Thus

$$\alpha_{(ep)}(\in) = \alpha^{EP}_{(ep)}(\in) + \alpha^D_{(ep)}(\in) + \alpha^A_{(ep)}(\in) \qquad (21)$$

Where

$$\alpha^{EP}_{(ep)}(\in) = \left(\frac{1}{4} \right)\left(\frac{\in}{\in_0} \right)\left[\frac{P_c(3\in^N + \in^C)^2 \Gamma^{EP}_{(ep)}(q,\in)}{(\in^2 - \overline{\in}_q^2)^2} \right] \qquad (22)$$

$$\alpha^{D}_{(ep)}(\in) = \left(\frac{1}{4} \right)\left(\frac{\in}{\in_0} \right)\left[\frac{P_c(3\in^N + \in^C)^2 \Gamma^D_{(ep)}(q,\in)}{(\in^2 - \overline{\in}_q^2)^2} \right] \qquad (23)$$

$$\alpha^{A}_{(ep)}(\in) = \alpha^{3A}_{(ep)}(\in) + \alpha^{4A}_{(ep)}(\in) \qquad (24)$$

$$\alpha_{(ep)}^{3A}(\in) = \left(\frac{1}{4}\right)\left(\frac{\in}{\in_0}\right)\left[\frac{P_c(3\in^N + \in^C)^2 \Gamma_{(ep)}^{3A}(q,\in)}{(\in^2 - \overline{\in}_q^2)^2}\right] \tag{25a}$$

$$\alpha_{(ep)}^{4A}(\in) = \left(\frac{1}{4}\right)\left(\frac{\in}{\in_0}\right)\left[\frac{P_c(3\in^N + \in^C)^2 \Gamma_{(ep)}^{4A}(q,\in)}{(\in^2 - \overline{\in}_q^2)^2}\right] \tag{25b}$$

The infrared absorption coefficient $\alpha_{(ep)}(\in)$ is proportional to the line width $\Gamma_{(ep)}(\in)$. The defect contribution to the absorption coefficient is dominant only force constant change parameter, which depends on the temperature and contribution of impurities. The anharmonic contribution $\alpha_{(ep)}^A(\in)$ to the absorption coefficient depends on the anharmonic parameter.

6 Absorption coefficients for high temperature superconductors

When in the close vicinity of T_c the superconductivity increases so that the cooper pairs dominate over normal pairs. In this case $(\in^C >> \in^N)$ and thus,

$$\alpha_{(ep)}^{EP}(k,C) = \frac{1}{4}\Omega_v P_c G_k^2 \left\{\left[-4\,\widetilde{\in}_k^2 \, (\in^C)^2 + \widetilde{\in}_k^4\right]\frac{n(\in^C)}{2} + 2\in_k^2 \, (\widetilde{\in}^C)^2 n_k\right\}$$
$$\times \frac{\in}{\in_0}\left[\frac{1}{(\in^C)^4} - \frac{2[\widetilde{\in}^2 \, (k,C) - \in^2]}{(\in^C)^6}\right] \tag{26a}$$

$$\alpha_{(ep)}^D(k,C) = 8\Omega_v P_c \sum_{k_1} G_k^2 |D(k_1, -k)|^2 \left[\frac{n(\in^C)}{2}\in_{k_1}\widetilde{\in}_{k_1} + (\widetilde{\in}^C)^2 n_{k_1}\right]\frac{\in}{\in_0}$$
$$\times \left[\frac{1}{(\in^C)^4} - \frac{2[\widetilde{\in}^2 \, (k,C) - \in^2]}{(\in^C)^6}\right] \tag{26b}$$

$$\alpha_{(ep)}^{3A}(k.C) = 18\Omega_v P_c \sum_{k_1,k_2} G_k^2 |V_3(k_1,k_2,-k)|^2 \eta_1 \left\{\left[S_{+\alpha}\,\widetilde{\in}_{+\alpha} + S_{-\alpha}\,\widetilde{\in}_{-\alpha}\right]\frac{n(\in^C)}{2}\right.$$
$$\left. + (\widetilde{\in}^C)^2 n_{k_1}n_{k_2}\right\}\left[\frac{1}{(\in^C)^4} - \frac{2[\widetilde{\in}^2 \, (k,C) - \in^2]}{(\in^C)^6}\right]\frac{\in}{\in_0} \tag{26c}$$

$$\alpha_{(ep)}^{4A}(k,C) = 96\Omega_v P_c \sum_{k_1,k_2,k_3} G_k^2 |V_4(k_1,k_2,k_3,-k)|^2 \left[\eta_2\left(S_{+\beta}\,\widetilde{\in}_{+\beta} + 3S_{-\beta}\,\widetilde{\in}_{-\beta}\right)\frac{n(\in^C)}{2}\right.$$
$$\left. + (\widetilde{\in}^C)^2 n_{k_1}n_{k_2}n_{k_3}\right]\left[\frac{1}{(\in^C)^4} - \frac{2[\widetilde{\in}^2 \, (k,C) - \in^2]}{(\in^C)^6}\right]\frac{\in}{\in_0} \tag{26d}$$

In the case of superconductors the contribution of normal energy becomes least than the cooper pairs energy, so that the cooper pairs energy is greater than the normal energy $(\in^N < \in^C)$. Therefore, the contributions of absorption coefficient can be written in this form

$$\alpha_{(ep)}^{EP}(k,q) = \frac{1}{4}\Omega_v P_c G_k^2 \left\{\left[-4\,\widetilde{\in}_k^2 \left((\in^C)^2 + 6\in^C\in^N\right) + \widetilde{\in}_k^4\right]\frac{n(\in^C)}{2} + 2\in_k^2 \left[(\widetilde{\in}^C)^2\right.\right.$$
$$\left.\left. + 6\,\widetilde{\in}^C\widetilde{\in}^N\right]n_k\left[\frac{1}{(\in^C)^4} - \frac{2[\widetilde{\in}^2 \, (k,q) - \in^2]}{(\in^C)^6}\right]\frac{\in}{\in_0}\right\} \tag{27a}$$

$$\alpha_{(ep)}^{D}(k,q) = 8\Omega_v P_c \sum_{k_1} G_k^2 \left|D(k_1,-k)\right|^2 \left\{ \in_{k_1} \widetilde{\in}_{k_1} \frac{n(\in^C)}{2} + [(\widetilde{\in}^C)^2 + 6\,\widetilde{\in}^C \widetilde{\in}^N]n_{k_1} \right\}$$

$$\times \frac{\in}{\in_0}\left[\frac{1}{(\in^C)^4} - \frac{2[\widetilde{\in}^2(k,q) - \in^2]}{(\in^C)^6} \right] \tag{27b}$$

$$\alpha_{(ep)}^{3A}(k.q) = 18\Omega_v P_c \sum_{k_1,k_2} G_k^2 \left|V_3(k_1,k_2,-k)\right|^2 \Big[\eta_1\big(S_{+\alpha}\,\widetilde{\in}_{+\alpha} + S_{-\alpha}\,\widetilde{\in}_{-\alpha}\big)\frac{n(\in^C)}{2}$$

$$+ [(\widetilde{\in}^C)^2 + 6\,\widetilde{\in}^C \widetilde{\in}^N]n_{k_1}n_{k_2} \Big]\left[\frac{1}{(\in^C)^4} - \frac{2[\widetilde{\in}^2(k,q) - \in^2]}{(\in^C)^6} \right]\frac{\in}{\in_0} \tag{27c}$$

$$\alpha_{(ep)}^{4A}(k,q) = 96\Omega_v P_c \sum_{k_1,k_2,k_3} G_k^2 \left|V_4(k_1,k_2,k_3,-k)\right|^2 \Big[\eta_2\big(S_{+\beta}\,\widetilde{\in}_{+\beta} + 3S_{-\beta}\,\widetilde{\in}_{-\beta}\big)\frac{n(\in^C)}{2}$$

$$+ [(\widetilde{\in}^C)^2 + 6\,\widetilde{\in}^C \widetilde{\in}^N]n_{k_1}n_{k_2}n_{k_3} \Big]\left[\frac{1}{(\in^C)^4} - \frac{2[\widetilde{\in}^2(k,q) - \in^2]}{(\in^C)^6} \right]\frac{\in}{\in_0} \tag{27d}$$

7 Temperature Dependences

The line-width depends on temperature, renormalized (or perturbed) mode energy via its constituents $\alpha_{(ep)}^{EP}(\in), \alpha_{(ep)}^{D}(\in)$ and $\alpha_{(ep)}^{A}(\in)$ of electron line-width and $\alpha_{(ep)}^{EP}(\in),\ \alpha_{(ep)}^{D}(\in)$ and $\alpha_{(ep)}^{A}(\in)$ of electron line-width. Substituting the value of electron line-width $\Gamma_{(ep)}(q,\in)$ in expressions of electron infrared absorption coefficients and is of the form

$$\alpha_{(ep)}^{EP}(q,\in) = (1/4)\Omega_v P_c G_k^2 \{[-4\,\widetilde{\in}_k^2(3\in^N + \in^C)^2 + \widetilde{\in}_k^4]N_{Q\sigma} + 2\{\in_k^2 n_k + (3\in^N + \in^C)4\in_k \tilde{n}_k$$

$$+ \frac{4\tilde{n}_k}{(3\in^N + \in^C)^2}\}(3\,\widetilde{\in}^N + \widetilde{\in}^C)^2\}\frac{\in}{\in_0}\left[\frac{1}{(\in^2 - \overline{\in}_q^2)^2} \right] \tag{28}$$

$$\alpha_{(ep)}^{D}(\in) = 8\Omega_v P_c \sum_{k_1} \left|D(k_1,-k)\right|^2 G_k^2 [\in_{k_1}\widetilde{\in}_{k_1}\ N_{Q\sigma} + n_{k_1}(3\,\widetilde{\in}^N + \widetilde{\in}^C)^2]\frac{\in}{\in_0}\left[\frac{1}{(\in^2 - \overline{\in}_q^2)^2} \right] \tag{29}$$

$$\alpha_{(ep)}^{3A}(\in) = 18\Omega_v P_c \sum_{k_1,k_2} G_k^2 \left|V_3(k_1,k_2,-k)\right|^2 \{\eta_1[S_{+\alpha}\,\widetilde{\in}_{+\alpha} + S_{-\alpha}\,\widetilde{\in}_{-\alpha}]N_{Q\sigma} + n_{k_1}n_{k_2}(3\,\widetilde{\in}^N + \widetilde{\in}^C)^2\}$$

$$\times \frac{\in}{\in_0}\left[\frac{1}{(\in^2 - \overline{\in}_q^2)^2} \right] \tag{30}$$

$$\alpha_{(ep)}^{4A}(q,\in) = 96\Omega_v P_c \sum_{k_1,k_2,k_3} G_k^2 \left|V_3(k_1,k_2,k_3,-k)\right|^2 \{\eta_2[S_{+\beta}\,\widetilde{\in}_{+\beta} + 3S_{-\beta}\,\widetilde{\in}_{-\beta}]N_{Q\sigma} + n_{k_1}n_{k_2}n_{k_3}$$

$$\times (3\,\widetilde{\in}^N + \widetilde{\in}^C)^2\}\frac{\in}{\in_0}\left[\frac{1}{(\in^2 - \overline{\in}_q^2)^2} \right] \tag{31}$$

In these above expressions we have seen that the electron infrared absorption coefficients are direct depends on the temperature through $N_{Q\sigma}$ and n_k (s). These coefficients are not only depends on the temperature but also depends on the energy and impurity concentration via renormalized mode energy and line-width $\Gamma_{(ep)}(q,\in)$. The electron line-width for the cubic and quartic anharmonicities shows the

temperature dependence of the form T^2 and T^3. Hence the infrared absorption coefficients show the strong temperature dependence of a crystal

8 Applications

The temperature dependence of $S_{\pm\alpha}$ and $S_{\pm\beta}$ are shown in figures (1) and (2). It is clear that $S_{-\alpha}$ and $S_{-\beta}$ is less compare to $S_{+\alpha}$ and $S_{+\beta}$ at low temperature. From these figures we can say that the anharmonicities are direct depends on the temperature through n_k (s).
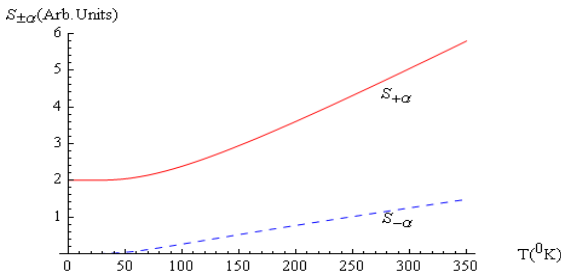

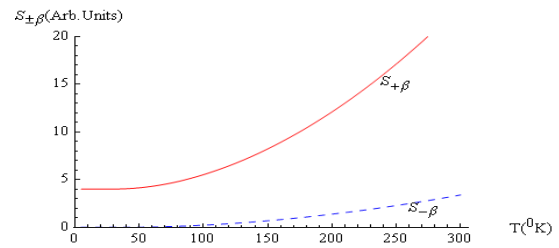**Figure 1:** Variation of $S_{+\alpha}$ and $S_{-\alpha}$ with

Figure 2: Variation of $S_{+\beta}$ and $S_{-\beta}$ with

The Cooper pair energy $\in^C$ and temperature T dependence of A is depicted in figure (3). This figure shows that A increases with temperature T and Cooper pair energy $\in^C$ as evidenced by this figure.
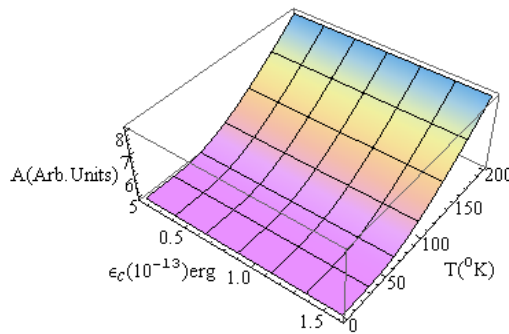

Figure 3: 3-D variation of A [Eq. (3.71)] with $\in^C$ and T

The variation of terms B, C and D with simultaneous change in cooper pair energy $\in^C$ and temperature T have been depicted in figures (4), (5) and (6). The effects of cooper pair energy $\in^C$, temperature T, electron-phonon interaction and defect on $\alpha_{(ep)}^{-1}(\in)\big|_{G.L.}$ are shown in figures (3), (4), (5) and (6).
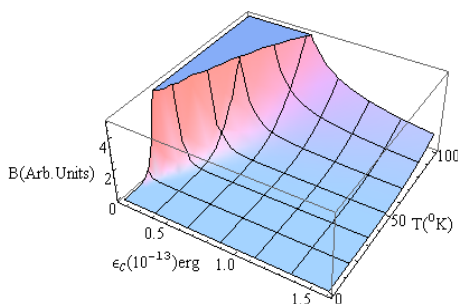

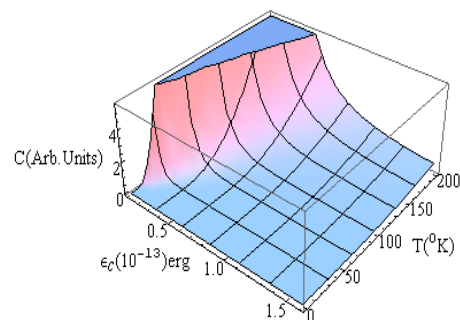**Figure 4:** 3-D variation of B [Eq. (3.72)] with $\in^C$

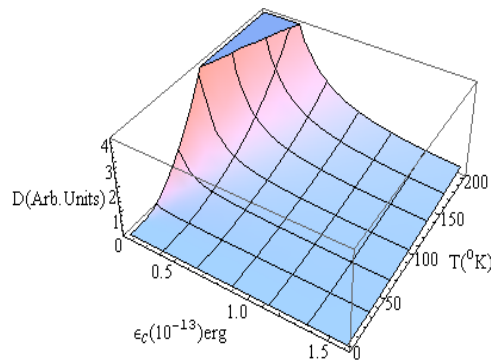**Figure 5:** 3-D variation of C [Eq. (3.73)] with $\in^C$ and T

Figure 6: 3-D variation of D [Eq. (3.74)] with $\in^C$ and T

Due to the non availability of several physical quantities for different samples and involvement of the wide range of energies, a great complexity in computation was eradicated by showing the general trends of the variation of absorption coefficient with changing energy, temperature and impurity concentration. This can be successfully explained with the help of present theory.

9 Conclusion

The absorption coefficient obtained here can be expressed as a sum of three terms for electronic absorption $\alpha_{(ep)}^{EP}(\in)$, $\alpha_{(ep)}^{D}(\in)$ and $\alpha_{(ep)}^{A}(\in)$. In the expressions of infrared absorption coefficients $\alpha_{(ep)}(\in)$, we have seen that the electron infrared absorption coefficients are direct depends on the temperature through $N_{Q\sigma}$ and n_k (s). These coefficients are not only depending on the temperature via renormalized mode energy and line-width $\Gamma_{(ep)}(q,\in)$. This theory is thus capable to study the absorption effects in High temperature Superconductors.

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In above calculations some physical parameters for A-15 (Nb_3Sn) superconductor are used, which are given in table.

M'	Mass of the impurity atom (Sn)	1.962×10^{-22} (gm)
M	Mass of the host atom (Nb$_3$)	4.607×10^{-22} (gm)
μ	Reduced mass	1.376×10^{-22} (gm)
V	Volume of the unit cell of Nb$_3$Sn crystal	1.480×10^{-22} (cm^3)
a_0	Lattice parameter of Nb$_3$Sn crystal	5.290×10^{-8} (cm)
α_1	Coefficient of interaction of two atoms of Sn	0.898×10^{8} (cm^{-1})
r_1	Equilibrium nearest-neighbor separation of Sn	6.490×10^{-8} (cm)
D_1	Dissociation energy of Sn molecule	5.030×10^{-12} (erg)
α_2	Coefficient of interaction of two atoms of Nb	0.753×10^{8} (cm^{-1})
r_2	Equilibrium nearest-neighbor separation of Nb	3.009×10^{-8} (cm)
D_2	Dissociation energy of Nb molecule	1.758×10^{-12} (erg)
v_p	Phonon velocity	5.0×10^{5} (cm/sec)
μ_{-2}	Second opposite moment of the frequency spectrum	3.165×10^{-15} (erg)
λ	Dimensionless parameter	5
η	Dimensionless parameter	3
N	Number of atoms per unit cell of the crystal	8